Momentum Distribution of a Dilute Unitary Bose Gas with Three-Body Losses

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Outline

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   - Virial expansion
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4. Conclusion

JILA's experiment

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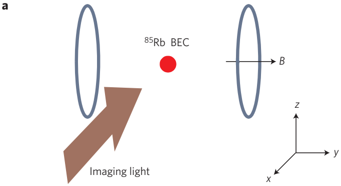
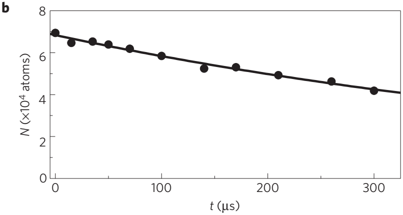
Universal dynamics of a degenerate unitary Bose gas

P. Makotyn, C. E. Klauss, D. L. Goldberger, E. A. Cornell* and D. S. Jin*



- $t = 0$ BEC (^{85}Rb), small $a\,(\approx 7\,\mathrm{nm})$, $T < 10\,\mathrm{nK}$
- $B \to B_0\,(a \to \infty)$ in $\Delta t = 5\,\mu\mathrm{s}$ *Unitary Limit*

JILA's experiment



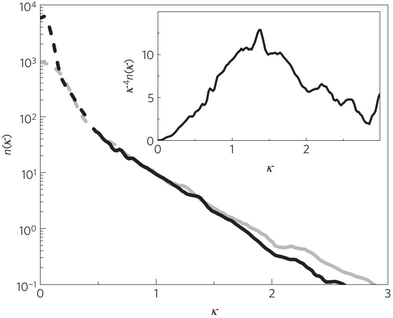


Figure 3 | The momentum distribution, $n(\kappa)$, plotted versus the scaled momentum, κ. Data for $\langle n \rangle = 5.5(3) \times 10^{12}$ cm^{-3} and $\langle n \rangle = 1.6(1) \times 10^{12}$ cm^{-3} are shown as the black and grey lines, respectively.

- Losses : $N(t) \searrow$
- "Universality" : rescaled momentum distribution

$$\langle n \rangle \equiv 6\pi^2\, k_n^3 \quad \kappa \equiv p/k_n \quad 1 = \frac{1}{2\,\pi^2} \int_0^{+\infty} \mathrm{d}\kappa\, \kappa^2\, n(\kappa)$$

Momentum distribution calculation

"High temperature" expansion : small parameter $n\lambda_{th}^3 \ll 1$
Two phenomena :

- 3-body losses
- Interactions

3 body losses

(F. Chevy, D. Petrov, C. Salomon, F. Werner ...)

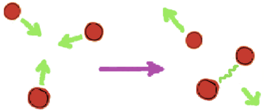


FIGURE – 3-body recombination-Courtesy L. Pricoupenko.

Use classical Boltzmann equation with 2-body elastic collisions and 3 body losses :

$$\partial_t f = I_{coll}[f] - \mathcal{L}_3[f]$$

$I_{coll}[f]$ is two-body elastic collision integral at unitarity.
$\mathcal{L}_3[f]$ is loss rate operator for unitary Bose gas :

$$\mathcal{L}_3[f](\mathbf{p}_1) = \int \mathrm{d}^3 p_2 \mathrm{d}^3 p_3 K_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) f(\mathbf{p}_1) f(\mathbf{p}_2) f(\mathbf{p}_3)$$

3 body losses

2-body collision rate (s^{-1}) (I_{coll}) $\gamma_2 \propto n$

3-body loss rate ($\mathcal{L}_3$) $\gamma_3 \propto n^2 \implies \gamma_3/\gamma_2 \propto n \ll 1$

3 body losses

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3-body loss rate ($\mathcal{L}_3$) $\gamma_3 \propto n^2 \implies \gamma_3/\gamma_2 \propto n \ll 1$

B. S. Rem *et al.*, PRL **110**, 163202 (2013) :

$\gamma_3/\gamma_2 = (1 - e^{-4\eta}) n \lambda_{th}^3$

η : "coupling" to deeply bound molecule

$\eta \geq 0$ (^{7}Li : $\eta = 0.2$, ^{85}Rb : $\eta = 0.06$).

$\implies$ **Treat $\mathcal{L}_3[f]$ perturbatively.**

3 body losses

Treat $\mathcal{L}_3[f]$ perturbatively.

$$\partial_t f = I_{coll}[f] - \mathcal{L}_3[f]$$

$f = f_0 + f_1 + \cdots$, f_0 : gaussian with *time dependent* energy(temperature) and particle number :

$f_0(p;t) = \frac{n(t)\lambda_{th}^3(t)}{h^3} e^{-\frac{p^2}{2\,m\,k_B\,T(t)}}$

$$I_{coll}[f_0] = 0$$
$$\partial_t f_0 = I'_{coll}[f_1] - \mathcal{L}_3[f_0]$$

Idea of the method :

- Eliminate term with f_1 by projection on the kernel of I'_{coll}, find differential equations for $n(t)$ and $T(t)$
- In order to find f_1, project onto space orthogonal to kernel (use gaussian$\times$ orthogonal polynomials).
  Order of magnitude : $f_1 \sim \mathcal{L}_3[f_0]/\gamma_2 \sim (n\lambda_{th}^3)\, f_0$.

3 body losses : results 1

- Rate equations :

$$\begin{aligned} \partial_t n &= -L_3\, n^3 \\ L_3 &\simeq 36\sqrt{3}\pi^2 \frac{\hbar^5}{m^3 (k_B\, T)^2}(1 - e^{-4\eta}) \\ \partial_t E &= -\frac{5}{9} E\, L_3\, n^2 \end{aligned} \quad (1)$$

- Temperature $T(t)$: $E = \frac{3}{2} k_B\, n(t)\, T(t)$.
  $n(t) \searrow$, $E(t) \searrow$ ... but $T(t) \nearrow$.
- Eq.(1) : L_3 obtained in B. S. Rem *et al.*, PRL **110**, 163202 (2013).

3 body losses : results 2

PRL **113,** 220601 (2014) PHYSICAL REVI

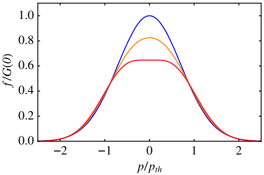


FIG. 1 (color online). Deformation of the momentum distribution of a unitary Bose gas due to three-body losses. From top to bottom: $n\lambda_{\mathrm{th}}^3(1-e^{-4\eta})=0$ (blue, Boltzmann gas), $n\lambda_{\mathrm{th}}^3(1-e^{-4\eta})=0.05$ (orange), and $n\lambda_{\mathrm{th}}^3(1-e^{-4\eta})=0.1$ (red).

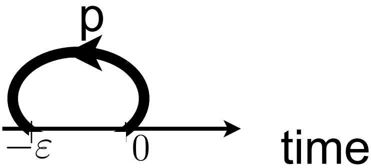
$$
\begin{aligned}
p_{th} &= \frac{\hbar}{\lambda_{th}} \\
f &= f_0 + f_1 \\
f_0(p,t) &= n(t)\frac{e^{-\frac{p^2}{2\,m\,k_B\,T(t)}}}{(2\pi\,m\,k_B\,T(t))^{3/2}} \\
f_1(p,t) &= \frac{\left(n(t)\lambda_{th}^3(t)\right)^2}{h^3}\,\xi(p/p_{th}(t)) \\
&\times\ (1-e^{-4\eta})
\end{aligned}
$$

Virial expansion-thermal equilibrium

- small parameter : fugacity $z = e^{\beta\mu} \ll 1$,

Virial expansion-thermal equilibrium

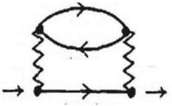
- small parameter : fugacity $z = e^{\beta\mu} \ll 1$, $z \simeq n\lambda_{th}^3$, $\lambda_{th} = \sqrt{\frac{2\pi\hbar^2}{k_B T m}}$.
- Expand $\rho_p = \langle c_{\mathbf{p}}^\dagger c_{\mathbf{p}} \rangle$ in powers of z.
- Diagrammatic approach :
  $\rho_p = -G(\mathbf{p}, \tau = 0^-)$.



Virial expansion-thermal equilibrium

Principle of the method :

- Feynman diagrams : building blocks are free particle (boson) propagators G^0 and coupling constant g.



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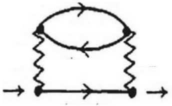
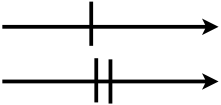


- Expand G^0 in power of fugacity :

$$
\begin{aligned}
G^0(\mathbf{p},\tau) &= e^{\mu\tau} \sum_{n\geq 0} G^{0,n}(\mathbf{p},\tau)\, z^n \\
G^{(0,0)}(\mathbf{p},\tau) &= -\Theta(\tau) e^{-\varepsilon_p \tau}, \ \underline{\text{retarded}} \\
G^{(0,n\geq 1)}(\mathbf{p},\tau) &= -e^{-\varepsilon_p \tau} e^{-n\,\beta\,\varepsilon_p} \\
\varepsilon_p &= \frac{p^2}{2\,m}
\end{aligned}
$$

High-temperature expansion-thermal equilibrium

$$G^{(0,0)}(\mathbf{p}, \tau) \quad = $$
$$G^{(0,1)}(\mathbf{p}, \tau) \quad = $$
$$G^{(0,2)}(\mathbf{p}, \tau) \quad = $$



Propagation of
a particle in **vacuum**

....

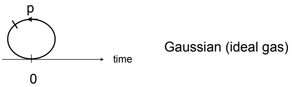
- A diagram with n slashes is of order z^n.
- $G^{(0,0)}$ can**not** got **backward** in (imaginary) time.

XL, PRA **84**, 053633 (2011)

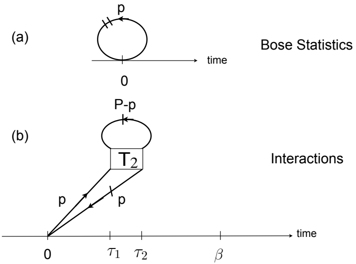
High-temperature expansion-thermal equilibrium

$\rho_p = \langle c_{\mathbf{p}}^{\dagger} c_{\mathbf{p}} \rangle = -G(\mathbf{p}, \tau = 0^{-})$

Order z



Order z^2



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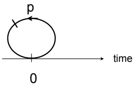
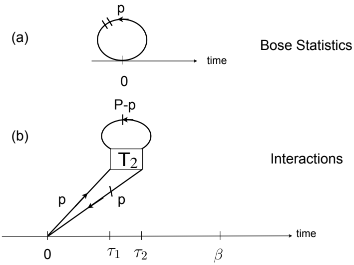


Gaussian (ideal gas)

Order z^2



$$\rho^{(2,b)}(p) = \frac{8\pi z^2}{m} \int_{\mathcal{C}_\gamma} \frac{\mathrm{d}s}{2\pi i} \int_0^{+\infty} \frac{\mathrm{d}P P^2}{2\pi^2} \frac{e^{-\beta s}}{\sqrt{-ms}}$$

$$\times \frac{e^{-\beta \frac{P^2}{4m}}}{\left[s + \frac{P^2}{4m} - \frac{p^2}{2m} - \frac{(P-p)^2}{2m}\right]\left[s + \frac{P^2}{4m} - \frac{p^2}{2m} - \frac{(P+p)^2}{2m}\right]}$$

Results-comparison to experiment

Uniform gas :

$$\rho(p) \quad = \quad z\, e^{-\beta \frac{p^2}{2\,m}} + z^2 \left[\rho_{\text{losses}}(p\lambda_{th}/\hbar) + \rho_{\text{virial}}(p\lambda_{th}/\hbar)\right]$$

Trapped gas :

Assume

- Thomas-Fermi $(1 - r^2/R^2)$ profile unchanged during ramping

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  $T(n(r); a^{-1} = 0; m; \hbar) = C\,\frac{\hbar^2\, n(r)^{2/3}}{m}$
  $\implies n(r)\lambda_{th}^3(r) = \text{constant}$

Results-comparison to experiment

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  $T(n(r); a^{-1} = 0; m; \hbar) = C\frac{\hbar^2\, n(r)^{2/3}}{m}$
  $\implies n(r)\lambda_{th}^3(r) = \text{constant} = f(z)$.
  Fugacity z is *uniform.*

Results-comparison to experiment

JILA's experiment parameters :

$$
\begin{aligned}
\langle n \rangle &\equiv 6\pi^2\, k_n^3 \\
\kappa &\equiv p/k_n \\
n(\kappa) &\propto \int_{r<R} d^3 r\, \rho(p; \mu(r); T(r)) \\
1 &= \frac{1}{2\,\pi^2} \int_0^{+\infty} \mathrm{d}\kappa\, \kappa^2\, n(\kappa)
\end{aligned}
$$

One parameter : fugacity z

Results-comparison to experiment

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$$\kappa \equiv p/k_n$$

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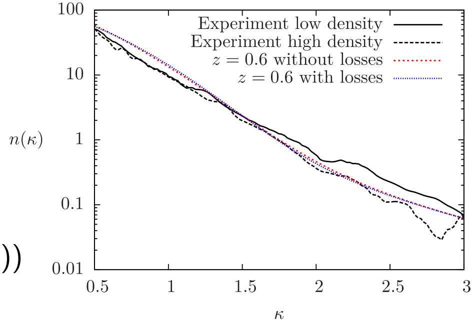

FIGURE – PRL **113**, 220601 (2014)

Best fit for $z \approx 0.6$.
Losses are weak here.

Conclusion

- Unitary ($a^{-1} = 0$) Bose gas with 3-body losses
- Controlled calculation if small parameter $z \simeq n\,\lambda_{th}^3 \ll 1$.
- Effects of 3-body losses and interaction/statistics $O(z^2)$.
- Comparison with experiment : $z \simeq 0.6$. Small ?
  $\simeq$ Ok for Equation of State of Unitary Fermi gas.
- Efimov physics ? Needs 3-body correlations ("T_3") :
  J. Hofmann and M. Barth PRA **93**, 061602 (2016) (good agreement with experiment if Efimov trimers states *not* populated).
- Project : losses for spin 1/2 fermions.
  3-body contact : $g^{(3)}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ if $R \to 0$.